Table 1. HCCN 1 facility-level patient characteristics, N (%)

Characteristics	Health centers 1 – 204, range
Total	1 – 43,398
Age	*
15-20 years	0 – 7,375 (0% - 100%)
21-44 years	0 – 36,023 (0% - 100%)
Race	*
Asian	0 – 1,021 (0% - 30.5%)
Native Hawaiian or other Pacific Islander	0 – 120 (0% - 4.3%)
Black/African American	0 – 5,171 (0% - 89.0%)
American Indian/Alaska Native	0 – 940 (0% - 98.2%)
White	0 – 27,023 (0% - 100%)
More than one race	0 – 1,641 (0% - 35.1%)
Unreported/Refused to report	0 – 18,126 (0% - 100%)
Ethnicity	
Hispanic/Latino	0 – 4,909 (0% - 54.7%)
Non-Hispanic/Latino	0 – 21,795 (0% - 100%)
Unreported/Refused to Report	0 – 21,548 (0% - 100%)

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Table 2. HealthEfficient facility-level patient characteristics, N (%)

Characteristics	Health center 1	Health center 2	Health center 3
Total	1,496	3,513	1,994
Age	*	*	*
15-20 years	229 (15.3%)	594 (16.9%)	441 (22.1%)
21-44 years	1,267 (84.7%)	2,919 (83.1%)	1,553 (77.9%)
Race	*	*	*
Asian	122 (8.2%)	96 (2.7%)	276 (13.8%)
Native Hawaiian or other Pacific Islander	8 (0.5%)	65 (1.9%)	20 (1.0%)
Black/African American	588 (39.3%)	1076 (30.6%)	642 (32.2%)
American Indian/Alaska Native	26 (1.7%)	75 (2.1%)	18 (0.9%)
White	588 (39.3%)	1403 (39.9%)	522 (26.2%)
More than one race	2 (0.1%)	21 (0.6%)	74 (3.7%)
Unreported/Refused to report	136 (9.1%)	777 (21.1%)	442 (22.2%)
Ethnicity	*	*	*
Hispanic/Latino	580 (38.8%)	0 (0%)	0 (0%)
Non-Hispanic/Latino	865 (57.8%)	1,654 (47.1%)	1,319 (66.2%)
Unreported/Refused to Report	51 (3.4%)	1,859 (52.9%)	675 (33.9%)

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Table 3. HCCN 1 clinician group/practice-level patient characteristics, N (%)

Characteristics	Sites 1 – 1,466, range
Total	1 – 7,475
Age	*
15-20 years	1 – 1,820 (0% - 100%)
21-44 years	1 – 6,392 (0% - 100%)
Race	*
Asian	0 – 839 (0% - 100%)
Native Hawaiian or other Pacific Islander	0 – 123 (0% - 0.1%)
Black/African American	0 – 2,821 (0% - 100%)
American Indian/Alaska Native	0 – 740 (0% - 100%)
White	0 – 5,876 (0% - 100%)
More than one race	0 – 561 (0% - 100%)
Unreported/Refused to report	0 – 3771 (0% - 100%)
Ethnicity	*
Hispanic/Latino	1 – 1,232 (0% - 100%)
Non-Hispanic/Latino	1 – 3,594 (0% - 100%)
Unreported/Refused to Report	1 – 4,708 (0% - 100%)

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Table 4. HealthEfficient clinician group/practice-level patient characteristics, N (%)

Characteristics	Sites 1 – 42, range
Total	1 – 1,448
Age	*
15-20 years	0 – 273 (0% - 100%)
21-44 years	0 – 1,223 (0% - 100%)
Race	*
Asian	0 – 230 (0% - 28.6%)
Native Hawaiian or other Pacific Islander	0 – 21 (0% - 9.1%)
Black/African American	0 – 579 (0% - 66.7%)
American Indian/Alaska Native	0 – 26 (0% - 50.0%)
White	0 – 583 (0% - 100%)
More than one race	0 – 48 (0% - 9.4%)
Unreported/Refused to report	0 – 297 (0% - 66.7%)
Ethnicity	*
Hispanic/Latino	0 – 572 (0% - 100%)
Non-Hispanic/Latino	0 – 972 (0% - 100%)
Unreported/Refused to Report	0 – 617 (0% - 100%)

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Table 5. Beta-binomial reliability estimates by age group, most or moderately effective methods

Level	Age group	Median N (all units)	Reliability (all units)	Median N (unit size ≥ 50)	Reliability (unit size ≥ 50)
Facility (HCCN 1)	15-44	1989	**0.953**	2170	**0.984**
Facility (HCCN 1)	21-44	1611	0.953	1748.5	0.980
Facility (HCCN 1)	15-20	363	0.908	430	0.963
Facility (HealthEfficient)	15-44	1994	**0.989**	1994	**0.989**
Facility (HealthEfficient)	21-44	1553	0.989	1553	0.989
Facility (HealthEfficient)	15-20	441	0.796	441	0.796
Clinician group/practice (HCCN 1)	15-44	148	**0.809**	373.5	**0.961**
Clinician group/practice (HCCN 1)	21-44	101	0.783	395	0.957
Clinician group/practice (HCCN 1)	15-20	30	0.744	139	0.951
Clinician group/practice (HealthEfficient)	15-44	29	**0.616**	304.5	**0.938**
Clinician group/practice (HealthEfficient)	21-44	20	0.598	260	0.944
Clinician group/practice (HealthEfficient)	15-20	4	0.464	74	0.876

Table 6. Beta-binomial reliability estimates by age group, LARC

Level	Age group	Median N (all units)	Reliability (all units)	Median N (unit size ≥ 50)	Reliability (unit size ≥ 50)
Facility (HCCN 1)	15-44	1989	**0.914**	2170	**0.955**
Facility (HCCN 1)	21-44	1611	0.910	1748.5	0.944
Facility (HCCN 1)	15-20	363	0.820	430	0.898
Facility (HealthEfficient)	15-44	1994	**0.979**	1994	**0.979**
Facility (HealthEfficient)	21-44	1553	0.976	1553	0.976
Facility (HealthEfficient)	15-20	441	0.902	441	0.902
Clinician group/practice (HCCN 1)	15-44	148	**0.691**	373.5	**0.893**
Clinician group/practice (HCCN 1)	21-44	101	0.660	395	0.884
Clinician group/practice (HCCN 1)	15-20	30	0.573	139	0.856
Clinician group/practice (HealthEfficient)	15-44	29	**0.490**	304.5	**0.845**
Clinician group/practice (HealthEfficient)	21-44	20	0.451	260	0.852
Clinician group/practice (HealthEfficient)	15-20	4	0.330	74	0.717

Table 7. Rates and reliabilities for most or moderately effective contraceptive method use and provision by facility, HealthEfficient, 2023.

Unit ID	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years
*	Most/Mod	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	Most/Mod	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	Most/Mod	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)
1	91	229	0.397	0.705	0.705	578	1267	0.456	0.986	0.986	669	1496	0.447	0.985	0.985
2	199	594	0.335	0.861	0.861	587	2919	0.201	0.994	0.994	786	3513	0.224	0.994	0.994
3	205	441	0.465	0.822	0.822	604	1553	0.389	0.988	0.988	809	1994	0.406	0.989	0.989
Total or Mean	**495**	**1264**	**0.392**	*	*	**1769**	**5739**	**0.308**	*	*	**2264**	**7003**	**0.323**	*	*
*	*	*	*	**Overall Reliability**	**Overall Reliability**	*	*	*	**Overall Reliability**	**Overall Reliability**	*	*	*	**Overall Reliability**	**Overall Reliability**
*	Median n	441	*	**0.796**	**0.796**	Median n	1553	*	**0. 989**	**0.989**	Median n	1994	*	**0. 989**	**0.989**
*	Min n	229	*			Min n	1267				Min n	1496			

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Table 8. Rates and reliabilities for LARC provision by facility, HealthEfficient, 2023.

Unit ID	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years
*	LARC	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	LARC	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	LARC	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)
1	5	229	0.022	0.852	0.852	62	1267	0.049	0.968	0.968	67	1496	0.045	0.971	0.971
2	42	594	0.071	0.937	0.937	150	2919	0.051	0.986	0.986	192	3513	0.055	0.988	0.988
3	53	441	0.120	0.917	0.917	231	1553	0.149	0.974	0.974	284	1994	0.142	0.978	0.978
Total or Mean	**100**	**1264**	**0.079**	*	*	**443**	**5739**	**0.077**	*	*	**543**	**7003**	**0.078**	*	*
*	*	*	*	**Overall Reliability**	**Overall Reliability**	*	*	*	**Overall Reliability**	**Overall Reliability**	*	*	*	**Overall Reliability**	**Overall Reliability**
*	Median n	441	*	**0.902**	**0.902**	Median n	1553	*	**0. 976**	**0.976**	Median n	1994	*	**0.979**	**0.979**
*	Min n	229	*			Min n	1267	*			Min n	1496	*		

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Table 9. Rates and reliabilities for most or moderately effective contraceptive method use and provision by clinician group/practice, HealthEfficient, 2023.

Unit ID	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years
*	Most/Mod	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	Most/Mod	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	Most/Mod	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)
1	0	1	0	0.077	NA	4	24	0.167	0.704	NA	4	25	0.16	0.682	NA
2	13	51	0.255	0.810	0.815	15	254	0.059	0.962	0.959	28	305	0.092	0.963	0.961
3	0	1	0	0.077	NA	0	3	0	0.229	NA	0	4	0	0.256	NA
4	2	14	0.143	0.539	NA	18	104	0.173	0.912	0.905	20	118	0.169	0.91	0.904
5	21	74	0.284	0.861	0.865	35	276	0.127	0.965	0.962	56	350	0.16	0.968	0.965
6	0	2	0	0.143	NA	0	3	0	0.229	NA	0	5	0	0.3	NA
7	8	51	0.157	0.810	0.815	57	328	0.174	0.97	0.968	65	379	0.172	0.97	0.968
8	0	1	0	0.077	NA	2	10	0.2	0.498	NA	2	11	0.182	0.486	NA

Unit ID	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years
*	Most/Mod	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	Most/Mod	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	Most/Mod	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)
9	6	36	0.167	0.750	NA	46	237	0.194	0.959	0.956	52	273	0.190	0.959	0.961
10	1	1	1	0.077	NA	2	12	0.167	0.543	NA	3	13	0.231	0.528	NA
11	90	225	0.400	0.949	0.951	576	1223	0.471	0.992	0.991	666	1448	0.460	0.992	0.904
12	1	4	0.250	0.250	NA	1	43	0.023	0.810	NA	2	47	0.043	0.801	0.966
13	0	0	NA	NA	NA	0	3	0	0.229	NA	0	3	0	0.205	NA
14	0	0	NA	NA	NA	1	1	1	0.090	NA	1	1	1	0.079	0.968
15	5	9	0.556	0.429	NA	9	30	0.300	0.748	NA	14	39	0.359	0.770	NA
16	0	0	NA	NA	NA	0	1	0	0.090	NA	0	1	0	0.079	0.956
17	12	22	0.545	0.647	NA	51	184	0.277	0.948	0.944	63	206	0.306	0.947	NA
18	2	2	1	0.143	NA	2	2	1	0.165	NA	4	4	1	0.256	0.991
19	16	31	0.516	0.721	NA	33	75	0.440	0.881	0.873	49	106	0.462	0.901	NA
20	15	40	0.375	0.769	NA	41	118	0.347	0.921	0.915	56	158	0.354	0.931	NA
21	1	7	0.143	0.369	NA	8	16	0.500	0.613	NA	9	23	0.391	0.664	NA
22	148	273	0.542	0.958	0.959	469	1137	0.412	0.991	0.99	617	1410	0.438	0.992	NA
23	1	8	0.125	0.400	NA	0	19	0	0.653	NA	1	27	0.037	0.699	NA
24	10	56	0.179	0.824	0.829	0	0	NA	NA	NA	10	56	0.179	0.828	0.943
25	0	0	NA	NA	NA	0	2	0	0.165	NA	0	2	0	0.147	NA
26	15	59	0.254	0.831	0.836	29	261	0.111	0.963	0.96	44	320	0.138	0.965	0.895
27	6	44	0.136	0.786	NA	31	260	0.119	0.963	0.96	37	304	0.122	0.963	0.927
28	1	6	0.167	0.333	NA	6	36	0.167	0.781	NA	7	42	0.167	0.783	NA
29	105	188	0.559	0.940	0.942	297	907	0.327	0.989	0.988	402	1095	0.367	0.989	0.991
30	0	2	0	0.143	NA	0	1	0	0.090	NA	0	3	0	0.205	NA
31	11	36	0.306	0.750	NA	0	0	NA	NA	NA	11	36	0.306	0.756	0.818
32	1	3	0.333	0.200	NA	8	33	0.242	0.766	NA	9	36	0.250	0.756	NA
33	0	2	0	0.143	NA	0	2	0	0.165	NA	0	4	0	0.256	0.962
34	2	5	0.4	0.294	NA	11	62	0.177	0.806	0.85	13	67	0.194	0.852	0.961
35	1	3	0.333	0.2	NA	5	26	0.192	0.720	NA	6	29	0.207	0.714	NA
36	0	0	NA	NA	NA	0	7	0	0.410	NA	0	7	0	0.375	NA
37	0	0	NA	NA	NA	0	2	0	0.165	NA	0	2	0	0.147	NA
38	1	2	0.5	0.143	NA	4	7	0.571	0.410	NA	5	9	0.556	0.436	NA
39	0	0	NA	NA	NA	0	3	0	0.229	NA	0	3	0	0.205	NA
40	0	4	0	0.250	NA	6	20	0.300	0.665	NA	6	24	0.250	0.673	NA
41	0	0	NA	NA	NA	1	6	0.167	0.373	NA	1	6	0.167	0.340	NA
42	0	1	0	0.077	NA	1	1	1	0.090	NA	1	2	0.50	0.147	NA
Total or Mean	**495**	**1264**	**0.392**	*	*	**1769**	**5739**	**0.308**	*	*	**2264**	**7003**	**0.323**	*	*
*	*	*	*	**Overall Reliability**	**Overall Reliability**	*	*	*	**Overall Reliability**	**Overall Reliability**	*	*	*	**Overall Reliability**	**Overall Reliability**
*	Median n	4	*	**0.464**	**0.876**	Median n	20	*	**0.598**	**0.944**	Median n	29	*	**0.616**	**0.938**
*	Min n	0	*			Min n	0	*			Min n	1	*		

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Table 10. Rates and reliabilities for LARC provision by clinician group/practice, HealthEfficient, 2023.

Unit ID	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years
*	LARC	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	LARC	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	LARC	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)
1	0	1	0	0.032	NA	0	24	0	0.440	NA	0	25	0	0.464	NA
2	2	51	0.039	0.628	0.598	4	254	0.016	0.893	0.881	6	305	0.020	0.914	0.89
3	0	1	0	0.032	NA	0	3	0	0.089	NA	0	4	0	0.122	NA
4	1	14	0.071	0.316	NA	4	104	0.038	0.773	0.753	5	118	0.042	0.803	0.758
5	5	74	0.068	0.710	0.683	8	276	0.029	0.900	0.89	13	350	0.037	0.924	0.903
6	0	2	0	0.062	NA	0	3	0	0.089	NA	0	5	0	0.148	NA
7	1	51	0.020	0.628	0.598	23	328	0.070	0.915	0.906	24	379	0.063	0.929	0.910
8	0	1	0	0.032	NA	0	10	0	0.246	NA	0	11	0	0.276	NA
9	0	36	0	0.543	NA	6	237	0.025	0.886	0.874	6	273	0.022	0.904	0.879
10	0	1	0	0.032	NA	0	12	0	0.282	NA	0	13	0	0.310	NA
11	5	225	0.022	0.882	0.868	62	1223	0.051	0.976	0.973	67	1448	0.046	0.980	0.975
12	0	4	0	0.117	NA	0	43	0	0.584	NA	0	47	0	0.619	NA
13	0	0	NA	NA	NA	0	3	0	0.089	NA	0	3	0	0.094	NA
14	0	0	NA	NA	NA	0	1	0	0.032	NA	0	1	0	0.033	NA
15	0	9	0	0.229	NA	1	30	0.033	0.495	NA	1	39	0.026	0.575	NA
16	0	0	NA	NA	NA	0	1	0	0.032	NA	0	1	0	0.033	NA
17	4	22	0.182	0.421	NA	19	184	0.103	0.857	0.843	23	206	0.112	0.877	0.846
18	1	2	0.5	0.062	NA	1	2	0.500	0.061	NA	2	4	0.500	0.122	NA
19	4	31	0.129	0.506	NA	11	75	0.147	0.710	0.687	15	106	0.142	0.786	0.738
20	4	40	0.1	0.569	NA	15	118	0.127	0.794	0.775	19	158	0.120	0.845	0.808
21	1	7	0.143	0.188	NA	4	16	0.250	0.343	NA	5	23	0.217	0.443	NA
22	34	273	0.125	0.900	0.888	181	1137	0.159	0.974	0.971	215	1410	0.152	0.980	0.974
23	0	8	0	0.209	NA	0	19	0	0.383	NA	0	27	0	0.483	NA
24	5	56	0.089	0.649	0.620	0	0	NA	NA	NA	5	56	0.089	0.660	0.598
25	0	0	NA	NA	NA	0	2	0	0.061	NA	0	2	0	0.065	NA
26	2	59	0.034	0.661	0.632	7	261	0.027	0.895	0.884	9	320	0.028	0.917	0.895
27	1	44	0.023	0.593	NA	7	260	0.027	0.895	0.884	8	304	0.026	0.913	0.89
28	0	6	0	0.166	NA	4	36	0.111	0.541	NA	4	42	0.095	0.593	NA
29	29	188	0.154	0.861	0.846	69	907	0.076	0.967	0.964	98	1095	0.089	0.974	0.967
30	0	2	0	0.062	NA	0	1	0	0.032	NA	0	3	0	0.094	NA
31	0	36	0	0.543	NA	0	0	NA	NA	NA	0	36	0	0.555	NA
32	0	3	0	0.090	NA	3	33	0.091	0.519	NA	3	36	0.083	0.555	NA
33	0	2	0	0.062	NA	0	2	0	0.061	NA	0	4	0	0.122	NA
34	0	5	0	0.142	NA	6	62	0.097	0.670	0.644	6	67	0.090	0.699	0.641
35	0	3	0	0.090	NA	3	26	0.115	0.459	NA	3	29	0.103	0.501	NA
36	0	0	NA	NA	NA	0	7	0	0.186	NA	0	7	0	0.195	NA
37	0	0	NA	NA	NA	0	2	0	0.061	NA	0	2	0	0.065	NA
38	1	2	0.500	0.062	NA	2	7	0.286	0.186	NA	3	9	0.333	0.238	NA

Unit ID	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	15 – 20 Years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	21 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years	15 - 44 years
*	LARC	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	LARC	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)	LARC	Total N	Rate	Reliability (all units)	Reliability (unit size≥50)
39	0	0	NA	NA	NA	0	3	0	0.089	NA	0	3	0	0.094	NA
40	0	4	0	0.117	NA	2	20	0.100	0.395	NA	2	24	0.083	0.454	NA
41	0	0	NA	NA	NA	0	6	0	0.164	NA	0	6	0	0.172	NA
42	0	1	0	0.032	NA	1	1	1	0.032	NA	1	2	0.500	0.065	NA
Total or Mean	**100**	**1264**	**0.079**	*	*	**443**	**5739**	**0.077**	*	*	**543**	**7003**	**0.078**	*	*
*	*	*	*	**Overall Reliability**	**Overall Reliability**	*	*	*	**Overall Reliability**	**Overall Reliability**	*	*	*	**Overall Reliability**	**Overall Reliability**
*	Median n	4	*	**0.330**	**0.717**	Median n	20	*	**0.451**	**0.852**	Median n	29	*	**0.490**	**0.845**
*	Min n	0	*			Min n	0	*			Min n	1	*		

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Table 11. HCCN 1 facility-level reliability, most or moderately effective method use and provision.

Item	Overall	Min	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	Max
Reliability (all units)	0.953	0.067	0.638	0.960	0.980	0.987	0.992	0.995	0.996	0.997	0.998	0.999	1
Reliability (unit size≥50)	0.984	0.854	0.925	0.973	0.983	0.989	0.993	0.995	0.996	0.997	0.998	0.999	1
N of Entities (all units)	204	1	21	21	20	20	24	23	23	19	15	18	1
N of Entities (unit size≥50)	193	1	20	20	19	23	15	21	23	21	13	18	1
N of Persons (all units)	703846	1	1557	7851	13883	21378	41115	59720	81383	89072	103090	284792	43398
N of Persons (unit size≥50)	703688	83	4012	10521	16136	30301	28985	55396	81383	100295	91867	284792	43398

Table 12. HCCN 1 facility-level reliability, LARC provision.

Item	Overall	Min	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	Max
Reliability (all units)	0.914	0.022	0.463	0.880	0.936	0.959	0.973	0.981	0.987	0.990	0.993	0.997	0.999
Reliability (unit size≥50)	0.955	0.647	0.798	0.919	0.948	0.964	0.977	0.983	0.987	0.990	0.994	0.997	0.999
N of Entities (all units)	204	1	21	20	21	21	19	22	27	13	20	20	1
N of Entities (unit size≥50)	193	1	20	19	19	20	22	18	21	19	21	14	1
N of Persons (all units)	703846	1	1557	7328	14411	22697	31509	53137	88376	60837	177821	246173	43398
N of Persons (unit size≥50)	703688	83	4012	9877	15805	24867	42033	48757	73506	85924	152734	246173	43398

Table 13. HCCN 1 clinician group/practice-level reliability, most or moderately effective method use and provision.

Item	Overall	Min	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	Max
Reliability (all units)	0.809	0.103	0.177	0.497	0.746	0.873	0.928	0.957	0.976	0.986	0.992	0.996	0.999
Reliability (unit size≥50)	0.961	0.849	0.879	0.915	0.942	0.959	0.972	0.981	0.987	0.991	0.994	0.997	0.999
N of Entities (all units)	1466	1	159	139	143	151	143	150	144	144	170	123	1
N of Entities (unit size≥50)	991	1	102	97	106	94	105	105	96	108	101	77	1
N of Persons (all units)	703846	1	313	1288	3881	9404	16383	30011	51441	86957	184265	319903	7475
N of Persons (unit size≥50)	696868	50	6685	9456	15548	19820	32940	49566	64002	104227	156101	238523	7475

Table 14. HCCN 1 clinician group/practice-level reliability, LARC provision.

Item	Overall	Min	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	Max
Reliability (all units)	0.690	0.033	0.062	0.236	0.474	0.675	0.793	0.869	0.922	0.954	0.972	0.987	0.996
Reliability (unit size≥50)	0.893	0.640	0.697	0.774	0.836	0.880	0.915	0.941	0.958	0.970	0.980	0.989	0.996
N of Entities (all units)	1466	1	159	139	143	151	141	147	148	152	142	144	1
N of Entities (unit size≥50)	991	1	102	97	100	100	98	99	99	108	89	99	1
N of Persons (all units)	703846	1	313	1288	3881	9404	16087	29024	52263	92735	147487	351364	7475
N of Persons (unit size≥50)	696868	50	6685	9456	14486	20882	30259	45062	63632	100296	126265	279845	7475

Table 15. HealthEfficient facility-level reliability, most or moderately effective method use and provision. (Due to a small number of entities at this level, we are only presenting the minimum and maximum reliability)

Item	Overall	Min	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	Max
Reliability (all units)	0.989	0.985	*	*	*	*	*	*	*	*	*	*	0.994
N of Entities (all units)	3	1	*	*	*	*	*	*	*	*	*	*	1
N of Persons (all units)	7003	1496	*	*	*	*	*	*	*	*	*	*	3513

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Table 16. HealthEfficient facility-level reliability, LARC provision. (Due to a small number of entities at this level, we are only presenting the minimum and maximum reliability)

Item	Overall	Min	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	Max
Reliability (all units)	0.979	0.971	*	*	*	*	*	*	*	*	*	*	0.988
N of Entities (all units)	3	1	*	*	*	*	*	*	*	*	*	*	1
N of Persons (all units)	7003	1496	*	*	*	*	*	*	*	*	*	*	3513

**Cells intentionally left empty*

Table 17. HealthEfficient clinician group/practice-level reliability, most or moderately effective method use and provision.

Item	Overall	Min	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	Max
Reliability (all units)	0.616	0.079	0.120	0.231	0.320	0.456	0.679	0.756	0.846	0.937	0.965	0.986	0.992
Reliability (unit size≥50)	0.938	0.818	0.831	0.895	0.915	0.943	0.959	NA*	0.964	0.968	0.990	NA*	0.991
N of Entities (all units)	42	1	5	6	2	4	4	5	4	4	4	4	1
N of Entities (unit size≥50)	15	1	2	1	2	1	3	NA*	2	1	3	NA*	1
N of Persons (all units)	7003	1	8	21	11	40	99	182	276	755	1279	4332	1448
N of Persons (unit size≥50)	6595	56	123	106	276	206	882	NA*	670	379	3953	NA*	1448

*There were only 8 deciles due to the distribution of reliability at this level.

Table 18. HealthEfficient clinician group/practice-level reliability, LARC provision.

Item	Overall	Min	Decile 1	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Decile 10	Max
Reliability (all units)	0.490	0.033	0.052	0.108	0.160	0.255	0.461	0.556	0.691	0.857	0.917	0.966	0.980
Reliability (unit size≥50)	0.845	0.598	0.619	0.738	0.783	0.846	0.885	0.890	0.899	0.910	0.970	0.975	0.975
N of Entities (all units)	42	1	5	6	2	4	4	5	4	4	4	4	1
N of Entities (unit size≥50)	15	1	2	1	2	1	2	1	2	1	2	1	1
N of Persons (all units)	7003	1	8	21	11	40	99	182	276	755	1279	4332	1448
N of Persons (unit size≥50)	6595	56	123	106	276	206	577	305	670	379	2505	1448	1448

Appendix A:

An Alternative Reliability Analysis Method for Assessing Quality Measures

Sam Field, PhD; Pat Malone, PhD; Philip Hastings, PhD; Eric Booth, MA

Introduction

We derive an alternative reliability parameter using health service quality indicators as an example. We argue that this formulation is more consistent with the underlying data-generating process than a commonly utilized beta-binomial approach from the health service quality literature (Adams, 2009). Our alternative approach is more widely applicable and can include situations where the quality measures for a health care provider are averaged over a small number of observations/patients. We describe a straightforward implementation of our approach using the R statistical software package.

The Beta-Binomial model

The measures of service quality we employ all take the form of a binomial proportion:

$$\frac{y_i}{n_i},$$

where

$$y_i$$

is a count of patients who were provided a particular service in cluster

$$i$$

, and

$$n_i$$

is the total number of patients in the cluster who were eligible to receive that service. The beta-binomial model begins with the assumption that the observed counts of services received by patients within cluster

$$i$$

arise as a binomial random variable with parameters

$$\pi_i$$

and

$$n_i$$

.

$$p(Y = y_i | n_i, \pi_i) = Binomial(n_i, \pi_i)$$

The approach further assumes that the cluster-level proportion parameters

$$\pi_i$$

, or "true" quality scores for providers, are sampled from a population of quality scores across clusters that follow a beta distribution with parameters

$$\alpha_0$$

and

$$\beta_0$$

.

$$p(\Pi = \pi_i | \alpha_0, \beta_0) = Beta(\alpha_0, \beta_0)$$

When a parameter of a random variable is itself a random variable, the statistical distribution of that parameter is known as a prior distribution. In this case, the beta distribution is a prior distribution for the parameter

$$\pi_i$$

in the binomial distribution. Furthermore, the substitution of

$$\alpha_0$$

and

$$\beta_0$$

for

$$\pi_i$$

in the binomial distribution leads to the beta-binomial distribution.

$$p(Y = y_i | n_i, \alpha_0, \beta_0) = BetaBinomial(n_i, \alpha_0, \beta_0)$$

Such a mixture of two statistical distribution does not always produce a third statistical distribution (i.e. beta-binomial) that is well-defined. When it does, the resulting distribution is called a "compound distribution" and the prior distribution for the parameter is known as a "conjugate prior". Thus, the beta distribution is a conjugate prior of the binomial distribution, and the beta-binomial distribution is the resulting compound distribution.

In practice, the parameters

$$\alpha_0$$

and

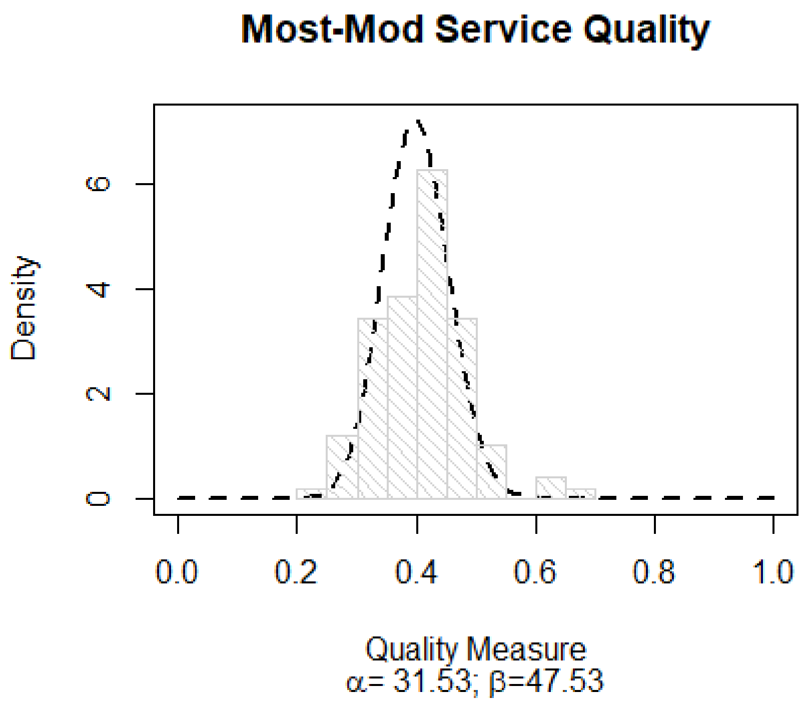
$$\beta_0$$

are estimated from the observed quality scores as

α_0 and β_0. As an example, we plot the density of the beta distribution with maximum likelihood estimates (MLE) α_0 and β_0 of α_0 and β_0 obtained from a beta-binomial regression of a sample of service quality measures taken from 99 U.S. counties. The measures obtained indicate the proportion of eligible patients in each county that received at least one “Most or Moderately Effective Contraceptive Method” (Most-Mod) service over the course of a year.

Figure 1 depicts the distribution of observed quality measures as a histogram, while the fitted beta distribution is depicted as a continuous density plot. The mean, median, and mode of the distribution is close to .4 indicating that in the average county, approximately 40% of women receive at least one Most-Mod service. In addition, most of the county-level variation is restricted to an interval of .2 to .6. As can be seen from the plot, the fit to the beta distribution is approximate. Specifically, the fitted distribution does not capture a small cluster of counties with quality scores > .6. As an approximation, however, the beta distribution does appear to fit the observed data adequately.

Figure 1: Histogram with density plot overlay depicting the county-level distribution of Most-Mod service quality measures.

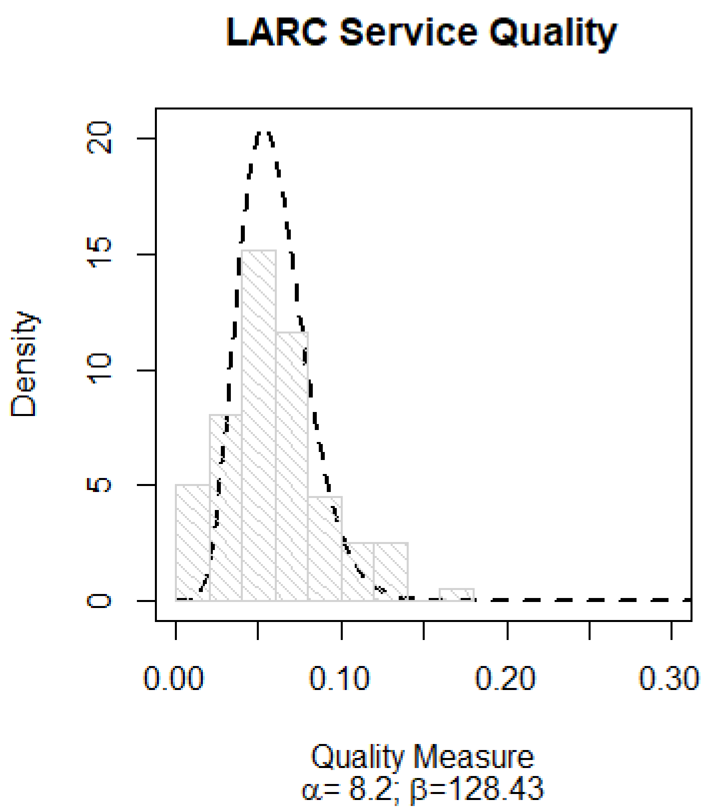


Although not evident in Figure 1, the range (i.e. x-axis) of the beta distribution is restricted to the 0,1 interval, which makes it a particularly suitable prior for a binomial parameter,

$$\pi_i$$

. This is easier to see if we plot a service quality indicator with an either very high or very low incidence. In Figure 2, we present the same plot for a service quality measure with a lower frequency, long-acting reversible contraception (LARC). In contrast to the estimated prior distribution for the Most-Mod service quality measure, the distribution of LARC quality measures is shifted considerably to the left with a noticeable right skew.

Figure 2: Histogram with density plot overlay depicting the county-level distribution of LARC service quality measures.



As Adams (2009) notes, the beta distribution is very flexible when it comes to fitting the observed distribution of service quality measures. Various combinations of the

$$\alpha_0$$

and

$$\beta_0$$

parameters generate a wide range of shapes - including U-shaped distributions where high and low-quality providers are widely separated from each other (Liu, et. al., 2013). Since this flexibility only requires the estimation of two parameters, the risk of over-fitting the observed data is minimal in cases where the number of clusters is large (e.g., > 30).

The predictive posterior distribution for health service quality

From a Bayesian perspective, predictions regarding the “true” cluster-level service quality score for any given cluster (e.g., provider or county) is based on the posterior predictive distribution (PPD) of

$$\pi_i$$

. The

PPD is the distribution of possible values for

$$\pi_i$$

for each cluster, conditional on the observed quality scores in that cluster,

$$\frac{y_i}{n_i}$$

, as well as the estimated parameters in the prior beta distribution (

$$\alpha_0$$

,

$$\beta_0$$

. Because of the conjugacy property discussed above, the PPD is analytically tractable. Specifically, the PPD for

$$\pi_i$$

is proportional to another beta distribution.

$$p(\Pi = \pi\_i \,|n\_i, y\_i, \alpha\_0, \beta\_0) \propto Beta((\alpha\_0 + y\_i), (\beta\_0 + (n\_i - y\_i))$$

In the current context, the PPD is the distribution of possible "true" cluster-level service quality scores based on sampling a single quality measure (i.e.,

$$\frac{y_i}{n_i}$$

)

from a population of true quality scores that are assumed to follow a beta distribution with known or estimated parameters

$$\alpha_0$$

and

$$\beta_0$$

.

For the purpose of deriving reliability measures, we focus on the mean,

$$E(\pi_i)$$

, of this distribution:

$$E(\pi\_i \mid n\_i, y\_i, \alpha\_0, \beta\_0) = E(Beta((\alpha\_0 + y\_i), (\beta\_0 + (n\_i - y\_i)))$$

We can substitute the analytically derived mean of the beta distribution,

$$\frac{\alpha_0}{(\alpha_o + \beta_0)}$$

, in the right side of the equation and simplify the result.

$$E(\pi_i | n_i, y_i, \alpha_0, \beta_0) = \frac{(\alpha_0 + y_i)}{(\alpha_0 + y_0) + \left(\beta_0 + (n_i - y_i)\right)} = \frac{(\alpha_0 + y_i)}{(\alpha_0 + \beta_0 + n_i)}$$

The empirical Bayes shrinkage estimator

Our derivation of the cluster-specific reliability estimate,

$$\lambda_i$$

, employs a commonly used identity for the empirical Bayes shrinkage estimator. Using this identity in the equation below (substituting empirically-estimated values

$$\alpha_0$$

and

$$\beta_0$$

for

unobserved population parameters

$$\alpha_0$$

and

$$\beta_0$$

, we

define the mean of the posterior predictive distribution as equal to

a

combination of the observed proportions,

$$\frac{y_i}{n_i}$$

, and

the mean of the prior beta distribution for the "true" service quality scores,

$$\pi_i$$

,

weighted by the reliability

$$\lambda_i$$

,

$$\frac{((\alpha_0 + y_i)}{(\alpha_0 + \beta_0 + n_i}$$

$$= \lambda_i \left(\frac{y_i}{n_i}\right) + \frac{(1 - \lambda_i)(\alpha_0}{(\alpha_0 + \beta_0}$$

As the reliability

$$\lambda_i$$

approaches 1 for a given cluster (nearing perfect reliability), the mean of the posterior predictive distribution for that cluster approaches the observed proportion. Conversely, for

$$\lambda_i$$

values less than 1, the mean of the predictive posterior is "shrunk" towards the [estimated] mean of the prior distribution

$$\frac{\alpha_0}{(\alpha_0 + \beta_0}.$$

For any given cluster, the more the shrinkage estimator is pulled towards the mean of the prior distribution, the less reliable the observed quality measures are for that cluster.

The classical test theory definition of reliability

In classical test score theory, reliability is defined as a ratio of true score variance to observed score variance (Novick, 1965):

$$\lambda_i = \frac{\sigma_{true}^2}{\sigma_{obs}^2}$$

In the beta-binomial model, the variance of the true score in the numerator is equal to the variance of the prior distribution for

$$\pi_i$$

. This is distributed as beta with an analytically derived variance:

$$\sigma_{true}^2 = var(\pi_i) = \frac{(\alpha_0\beta_0)}{(\alpha_0 + \beta_0)^2 * (\alpha_0 + \beta_0 + 1)}$$

The variance in the observed service incidence,

$$y_i$$

, is the variance of the compound distribution - the beta-binomial. However, we need to derive the variance of the observed proportions,

$$\frac{y_i}{n_i}$$

. In the first step, we note that the variance of any random variable multiplied by a constant is the variance of the random variable times the square of the constant. Thus,

$$\sigma_{obs}^2 = var\left(\frac{y_i}{n_i}\right)$$

$$= var(y_i) \times \left(\frac{1}{n_i}\right)^2$$

In the second step we replace

$$var(y_i)$$

with the analytically derived variance of beta-binomial distribution. Thus,

$$\sigma_{obs}^2 = var(y_i) \times \left(\frac{1}{n_i}\right)^2$$

$$= \left(\frac{n_i(\alpha_0\beta_0)(\alpha_0 + \beta_0 + n_i)}{(\alpha_0 + \beta_0)^2(\alpha_0 + \beta_0 + 1)}\right)$$

$$\times \left(\frac{1}{n_i}\right)^2$$

In the final step, the ratio of true score to observed score variance in the beta-binomial model,

$$\lambda_i$$

becomes:

$$\lambda_i = \frac{\sigma_{true}^2}{\sigma_{obs}^2}$$

$$= \frac{(\alpha_0\beta_0)}{(\alpha_0+\beta_0)^2 * (\alpha_0+\beta_0+1)} \Bigg/ \left(\frac{n_i(\alpha_0\beta_0)(\alpha_0+\beta_0+n_i)}{(\alpha_0+\beta_0)^2(\alpha_0+\beta_0+1)}\right) \times \left(\frac{1}{n_i}\right)^2$$

After simplifying we are left with a straight-forward expression for

$$\lambda_i$$

.

$$\lambda_i = \frac{n_i}{\alpha_0+\beta_0+n_i}$$

Algebraic proof of the identity above is available from the authors upon request. [1]

To illustrate the PPD and the empirical Bayes shrinkage estimator, we return to the 99-county example and the Most-Mod quality measures. In Figure 3, the PPDs for 2 different counties are plotted on top of the fitted prior distribution previously seen in Figure 1. We also indicate the means of each PPD as dotted vertical lines and of the observed quality measure (

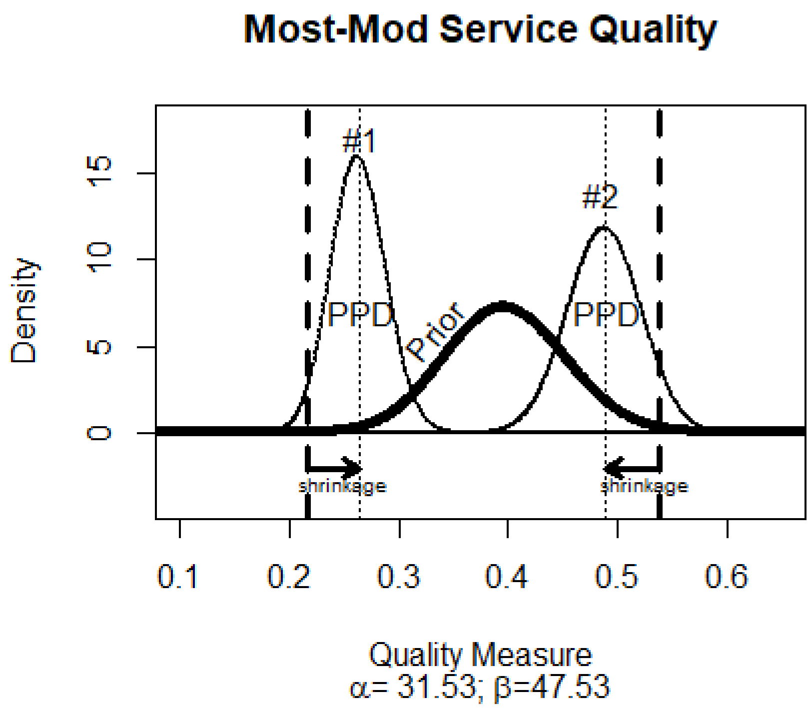
$$\frac{y_i}{n_i}$$

) as

[1] Carlin and Louis (2000) also derive an expression for the beta-binomial reliability parameter. Although their derivation is based on a different parameterization of the beta prior distribution, when expressed as a function of α_0 and β_0, their result is equivalent to ours (pp. 67-68).

thick, dashed vertical lines. The arrows indicate the direction and magnitude of the shrinkage of the EB estimate towards the mean of the prior distribution.

Figure 3: PPD for two counties with a prior distribution overlay.



Although the general shapes of the plotted PPDs are similar, the more peaked distribution seen for the county on the left (county #1) reflects greater certainty about the location of that county's "true" quality score. As our formulation of reliability indicates, this difference between the two counties is entirely a function of their difference in size. The numbers of patients in the left and right clusters are 230 and 141, respectively.

An alternative expression for Beta-Binomial reliability

In the sections above, we have derived a formulation for reliability that is consistent with the definition of the Bayesian shrinkage estimator as the mean of the posterior predictive distribution of the beta-binomial model. It is also consistent with the classical test theory, which views reliability as the proportion of total measurement variance that is attributable to true score variance. However, as we discuss below, it does not appear to be consistent with a formulation of beta-binomial reliability that often appears in the health care quality literature.

In a widely cited technical report from the RAND Corporation that was written for health care quality researchers, Adams (2009) offered an alternative formulation for beta-binomial reliability. Their approach was based on a least-squares formulation for reliability (Raudenbush & Bryk, 2002). Specifically,

$$\lambda_i = \frac{var(\pi_i)}{\left(var(\pi_i) + \frac{var\left(\frac{y_i}{n_i}\right)}{n}\right)}$$

For

$$var(\pi_i)$$

,
Adams used the variance of the prior beta distribution.

$$var(\pi_i) = \frac{(\alpha_0 \beta_0)}{(\alpha_0 + \beta_0)^2 * (\alpha_0 + \beta_0 + 1)}$$

,

while

$$var\left(\frac{y_i}{n_i}\right)$$

was
set equal to the variance of the binomial distribution,

$$var\left(\frac{y_i}{n_i}\right) = \frac{y_i}{n_i} \times \left(1 - \frac{y_i}{n_i}\right)$$

Thus, the Adams formulation for beta-binomial reliability is:

$$\lambda_i = \frac{\left(\frac{(\alpha_0\beta_0)}{(\alpha_0+\beta_0)^2 * (\alpha_0+\beta_0+1)}\right)}{\left(\frac{(\alpha_0\beta_0)}{(\alpha_0+\beta_0)^2 * (\alpha_0+\beta_0+1)} + \frac{\left(\frac{y_i}{n_i} \times \left(1 - \frac{y_i}{n_i}\right)\right)}{n_i}\right)}$$

To demonstrate the practical consequences of the two different approaches to calculating beta-binomial reliability, we return to the 99-county example and the Most-Mod quality measure. In Figure 4, we have plotted county-level reliability parameters calculated using the formulation that we described against the results obtained under Adams' (2009) approach. The diagonal represents the line of equality. The appearance of the plotting characters varies along two dimensions. The size of the characters is proportional to the natural log of the cluster size, ln(n_i), while character type identifies counties with observed measures ($\frac{y_i}{n_i}$) that are either within or outside the interquartile range of the sample.

Figure 4: Comparison of reliability parameters: Most-Mod quality measure.

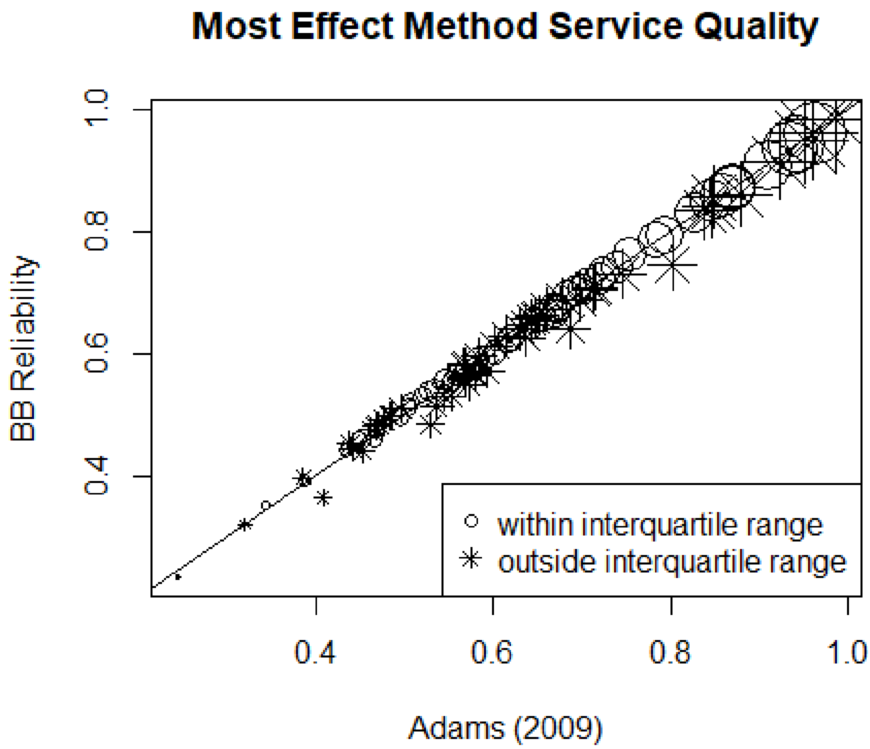
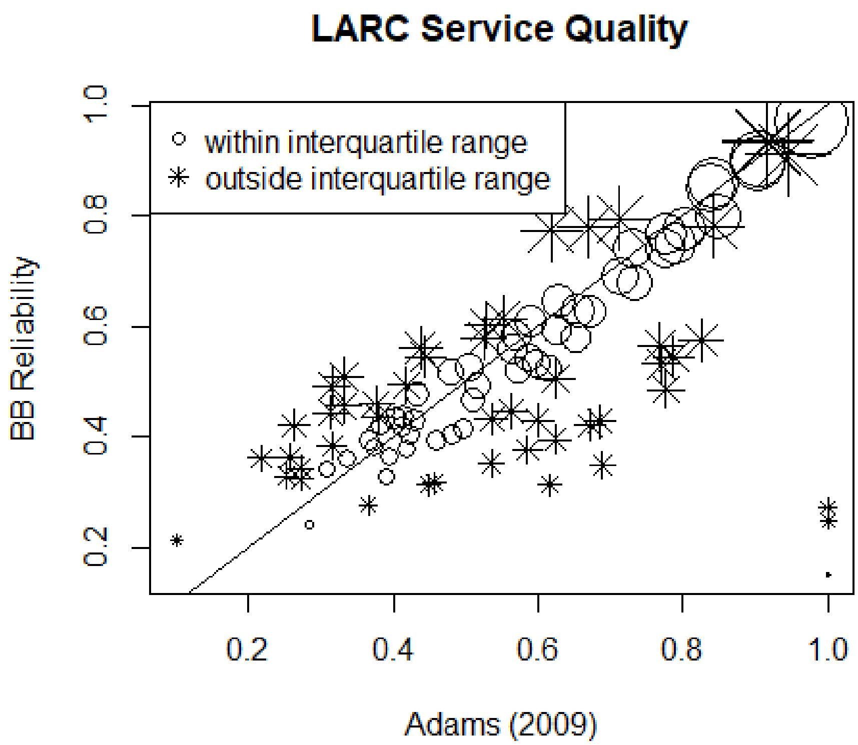


Figure 4 reveals very little difference between the two approaches. However, when we turn to the same plot for the LARC quality measure shown in Figure 5, substantial differences between the two methodologies emerge. Specifically, the disagreement between the methods appears greater in counties where the observed quality measure lies outside versus inside the inter-quartile range of the sample.

This pattern of results is expected because the reliability calculations that we describe depend *exclusively* on the size of the cluster while the Adams (2009) approach depends on both the size and the observed incident count. The results were not as evident with the Most-Mod quality measure because the binomial variance is relatively insensitive to variation in the quality measures within the central range of the 0,1 interval.

Figure 5: Comparison of reliability parameters: LARC quality measure.



When the variation in the cluster-level quality measures moves toward the boundaries, the between-cluster differences in the binomial variance component of the reliability parameter get much larger, and identically sized clusters can have dramatically different reliabilities. This is not possible with our calculations as identically sized clusters sampled from the same prior distribution will have equal reliability. It is also worth noting that the three small counties that appear in the bottom-right corner of the plot had perfect reliability under the Adams (2009) approach. This is because the variance of the binomial distribution equals zero when the observed measure is either 0 or 1.

Although the Adams (2009) formulation has been used to calculate beta-binomial reliability in previous empirical studies of health care quality (e.g., Adams and Paddock, 2017; Blair, et. al., 2015; Kazis, et. al., 2017; Staggs and Cramer, 2016), it has two distinct disadvantages when compared to the approach described here. First, we argue from statistical principle that the variance of the observed quality measures should be based on the beta-binomial compound distribution, and not the sum of the prior distribution (beta) and likelihood distribution (binomial) variances. Thus, the reliability formulation offered by Adams does not appear to be consistent with the underlying statistical model.

Second, under the Adams approach a component of measurement variance is determined by the observed data itself - the score,

$$\frac{y_i}{n_i}$$

. For clusters in which the observed measures equal 0 or 1, the binomial variance used in the calculations will equal zero, and the reliability measure will, consequently, be unstable in small clusters. The reliability calculations are also unstable when the bulk of the cluster-level variation lies close to the 0,1 boundaries - a situation that can be present even when the clusters are large. Indeed, Figure 5 demonstrates that the Adams (2009) approach can produce both substantially inflated and deflated estimates of reliability under such conditions. In contrast, the formulation that we offer does not depend on

$$y_i$$

and is equally valid across all cluster sizes.

Implementation

The approach to reliability calculation that we advocate is mathematically straightforward and can be implemented in many statistical software packages. In our implementation of the method, we use the statistical software R; specifically, we use the *vglm()* function in the "*VGAM"* package (Yee and Mohler, 2020). The procedure reparametrizes the beta-binomial distribution from

$$\text{betabinomial}(n_i \alpha_0, \beta_0)$$

to

$$\text{betabinomial}(\mathrm{n_i}, \mu, \gamma)$$

, where

$$\mu = \frac{\alpha_0}{(\alpha_0 + \beta_0)}$$

, or the mean of the prior distribution, and

$$\gamma = \frac{1}{(\alpha_0 + \beta_0 + 1)}$$

. The second parameter,

$$\gamma$$

, is interpreted as the overdispersion parameter or intra-cluster correlation coefficient (ICC), and it is possible to write reliability as a function of

$$\gamma$$

:

$$\lambda_i = \frac{n_i}{\left(\frac{1}{\hat{\gamma}} + (\mathrm{n}_i - 1)\right)}$$

.

As the ICC coefficient,

$$\gamma$$

, approaches 1, the reliability parameter also approaches 1, regardless of the within-cluster sample size.

The mean of the beta distribution,

$$\mu$$

, is linked to a linear combination of covariates via the inverse logit link.

$$\mu = e^{Xb} / (1 + e^{Xb})$$

,

Where X is a matrix of covariates (including a constant) and *b* is a vector of regression parameters. In our implementation, we exclude covariates and estimate an intercept-only model. The ICC is also estimated on the logit scale.

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R syntax to calculate beta-binomial reliability

```
# This is a very sparse R function that calculates reliability measures for binomial count
# data assuming a beta-binomial distribution.  The arguments in the function "y" and "n"
# correspond to the total number of "successes" and "trials" respectively.

beta_rel <- function(y,n){

  fit <- vglm(cbind(y,n-y) ~ 1, betabinomial)  #estimate beta-binomial model
  parms <- coef(fit, matrix = TRUE) #extract logit(mu) and logit(gamma)

  #Inverse logit link
  mu <- exp(coef(fit, matrix = TRUE)[1])/(1+exp(coef(fit, matrix = TRUE)[1]))
  gamma <-exp(coef(fit, matrix = TRUE)[2])/(1+exp(coef(fit, matrix = TRUE)[2]))
  theta <- gamma/(1-gamma)

  #Derive Beta parameters
  alpha=mu/theta;
  beta=(1-mu)/theta;

  #Calculate reliability
  rel <- n/(alpha+beta+n)

  cbind(y,n,id,rel)
}
```